The minimum value for the specific heat of water has been placed at various temperatures by different observers. Regnault found no minimum value; other observers have placed it as follows:—

Bartoli and Stracciati	20°	Rowland	30°
Lüdin	25°	Barnes	38°

Our value for the minimum agrees with that of Lüdin.

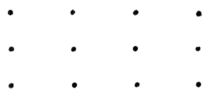
Memoir on the Theory of the Partitions of Numbers. Part V.—*Partitions in Two-dimensional Space.*

By Major P. A. MacMahon, D.Sc., F.R.S.

(Received December 31, 1910,—Read January 26, 1911.)

(Abstract.)

This paper is principally concerned with the enumerations of the partitions of numbers in two-dimensional space. Such partitions are such that the parts are placed at the points or nodes of a lattice, rectangular in shape, and



having given numbers of nodes in the sides, and the parts of the numbers partitioned are placed at the nodes in such wise that descending order of magnitude in each row of parts is from left to right and in each column of parts from top to bottom.

If the nodes in the sides be m, n, respectively, and the part magnitude be restricted so as to be not greater than l, the generating function which gives by the coefficients of x^w in the ascending expansion the number of such partitions of the number w has been long conjectured by the author to have the expression

$$\frac{\begin{array}{l}(l+1)(l+2)\dots(l+m)\\ \qquad\quad (l+2)\dots(l+m+1)\\ \qquad\qquad\quad \ddots \qquad\quad \ddots\\ \qquad\qquad\qquad (l+n)\dots(l+m+n-1)\end{array}}{\begin{array}{l}(1)(2)\dots(m)\\ \qquad (2)\dots(m+1)\\ \qquad\quad \ddots \qquad \ddots\\ \qquad\qquad (n)\dots(m+n-1)\end{array}}$$

where (s) denotes $1-x^s$; but previous attempts to establish this had, except in certain particular cases, resulted in failure.

The present communication shows that the generating function may be written

$$\sum_{s=0}^{s=(m-1)(n-1)} \frac{(l-s+1)\ldots(l-s+mn)\,L_s}{(1)\ldots(mn)},$$

where the functions L_s are readily derivable from any given lattice, and the only question thereafter is the examination of these functions and the simplification of the expression for the generating function.

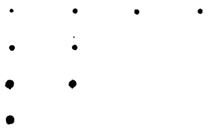
Considerable progress is made with these matters, and it is shown that if the conjectured form be true when l does not exceed mn, it must be true in general.

Further, that if the simplified form be assumed to be composed entirely of powers of factors

$$(s),\ (l+s),$$

where s is free from l, the form must be that which has been conjectured.

The method is equally valid for partitions at the points of an incomplete lattice in which the rows do not contain the same numbers of nodes, *e.g.*,



and a further study of these is necessary before the main result can be considered as established with complete rigour. This will form the subject of another communication to the Society.